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# LOEX Option: A Combination of Exchange and Lookback Options

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## Abstract

This paper studies a path-dependent derivative obtained by combining exchange and lookback features, called the LOEX option. The terminal payoff is the positive part of the running maximum of the first asset minus the running minimum of the second asset. A two-asset Black–Scholes framework is formulated with the running extrema included as state variables. The interior pricing partial differential equation is stated, and a meshfree radial-basis-function discretization is described for fixed extrema. Numerical illustrations compare the reduced-state LOEX surface with a standard exchange-option surface. Several mathematical inconsistencies in the supplied formulation are corrected, including the exchange-payoff orientation, the normal distribution function, the Itô differential, the extrema in the conditional expectation, and the radial-basis centers. Because the supplied numerical files do not include MATLAB code, convergence tests, or a full discretization of the extrema variables, the reported plots should be regarded as exploratory rather than as a validated market-pricing implementation.

**Keywords:** Exchange option, Lookback option, Path dependence, Black–Scholes model, Meshfree method.

## 1| Introduction

Derivative pricing contains many contracts whose payoff depends on more than a terminal asset value. This paper considers a LOEX option, constructed by combining exchange and

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lookback features. The contract depends on the running maximum of one asset and the running minimum of another, so its value is path dependent.

The background is divided into exchange-option and lookback-option pricing. **Exchange options.** Margrabe introduced the classical option to exchange one asset for another [1]. Subsequent studies used changes of numeraire, Mellin transforms, stochastic volatility, credit risk, and jump-diffusion models [2, 3, 4, 5, 6, 7].

**Lookback options.**

Early lookback-option studies derived formulas for floating- and fixed-strike contracts [8, 9]. Later work considered quanto lookback options, double lookbacks, and Mellin-transform solutions [10, 11, 12].

The proposed LOEX payoff combines these two strands. Its potential use as a hedging instrument would require empirical calibration, liquidity analysis, and comparison with portfolios of traded contracts.

Section 2 reviews the component contracts. Section 3 formulates the LOEX payoff and numerical scheme. Section 4 presents the supplied numerical illustration, and Section 5 concludes.

# 2 | Preliminary Concepts

Work on a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, Q)$ under a risk-neutral measure $Q$. Let the two non-dividend-paying assets satisfy

$$\begin{aligned} dS_t &= rS_t\,dt + \sigma_1 S_t\,dW_t^1, \\ dS_t^* &= rS_t^*\,dt + \sigma_2 S_t^*\,dW_t^2, \end{aligned} \tag{1}$$

where $r$ is the constant risk-free rate, $\sigma_1, \sigma_2 > 0$ are volatilities, and $d\langle W^1, W^2\rangle_t = \rho\,dt$ with $-1 \leq \rho \leq 1$.

## 2.1 | Exchange Option

An exchange option gives its holder the right, but not the obligation, to exchange the second asset for the first at maturity. If $C(t, S, S^*)$ denotes its value, the payoff is

$$C(T, S_T, S_T^*) = (S_T - S_T^*)^+. \tag{2}$$

Now, using the Feynman–Kac formula:

$$\frac{\partial C}{\partial t} + rS\frac{\partial C}{\partial S} + rS^*\frac{\partial C}{\partial S^*} + \frac{1}{2}\sigma_1^2 S^2\frac{\partial^2 C}{\partial S^2} + \rho\sigma_1\sigma_2 SS^*\frac{\partial^2 C}{\partial S\,\partial S^*} + \frac{1}{2}\sigma_2^2(S^*)^2\frac{\partial^2 C}{\partial (S^*)^2} - rC = 0. \tag{3}$$

with terminal condition

$$C(T, S, S^*) = (S - S^*)^+. \tag{4}$$

The closed formula for the Exchange option according to PDE (3) is as follows [5]

$$C(t, S_1, S_2) = S_1\Phi(d) - S_2\Phi(d - \sigma\sqrt{T-t}), \tag{5}$$

where

$$\begin{aligned} d &= \frac{ln\left(\frac{S_1}{S_2}\right)}{\sigma\sqrt{T-t}} + \frac{\sigma\sqrt{T-t}}{2}, \\ \sigma &= \sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}, \\ \Phi(x) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-u^2/2}\, du. \end{aligned} \tag{6}$$

## 2.2 | Lookback Option

The Lookback option is an exotic option that depends on the asset price path. In general, the Lookback option is divided into four categories. In this section, we summarize everything and present the price equation. Consider the following symbols:

$$\begin{aligned} M(S(t),t) &= \max_{0\le\omega\le t} S(\omega), \\ m(S(t),t) &= \min_{0\le\omega\le t} S(\omega). \end{aligned} \tag{7}$$

Now we have Lookback options and their price equations [13]

**Floating strike Lookback call option**

$$\begin{aligned} &payoff \to S(T) - m(S(T),T) \\ &Eq. \to S\Phi\left(\frac{\ln\left(\frac{S}{m(S,t)}\right)+\left(r+\frac{\sigma_1^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}\right) - m(S,t)e^{-r(T-t)}\Phi\left(\frac{\ln\left(\frac{S}{m(S,t)}\right)+\left(r-\frac{\sigma_1^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}\right) \\ &-\frac{S\sigma_1^2}{2r}\Phi\left(\frac{\ln\left(\frac{m(S,t)}{S}\right)-\left(r+\frac{\sigma_1^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}\right) + e^{-r(T-t)}\frac{S\sigma_1^2}{2r}\Phi\left(\frac{\ln\left(\frac{m(S,t)}{S}\right)+\left(r-\frac{\sigma_1^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}\right). \end{aligned} \tag{8}$$

**Floating strike Lookback put option**

$$\begin{aligned} &payoff \to M(S(T),T) - S(T) \\ &Eq. \to -S\Phi\left(-\frac{\ln\left(\frac{S}{M(S,t)}\right)+\left(r+\frac{\sigma_1^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}\right) + M(S,t)e^{-r(T-t)}\Phi\left(-\frac{\ln\left(\frac{S}{M(S,t)}\right)+\left(r-\frac{\sigma_1^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}\right) \\ &+\frac{S\sigma_1^2}{2r}\Phi\left(\frac{\ln\left(\frac{S}{M(S,t)}\right)+\left(r+\frac{\sigma_1^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}\right) - e^{-r(T-t)}\frac{S\sigma_1^2}{2r}\left(\frac{M}{S}\right)^{\frac{2r}{\sigma_1^2}}\Phi\left(\frac{\ln\left(\frac{S}{M(S,t)}\right)-\left(r-\frac{\sigma_1^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}\right). \end{aligned} \tag{9}$$

**Fixed strike Lookback call option**

$$payoff \to MAX(M(S(T),T) - K, 0)$$

At the time t, the price would be $e^{-r(T-t)}E[\max(\max(M_0^t(S(t),t), M_t^T(S(T),T)) - K, 0)]$,

$$\begin{aligned} &Eq. \to \begin{cases} H(t,S;K) & M_0^t(S(t),t) \le K \\ e^{-r(T-t)}(M_0^t(S(t),t) - K) + H(t,S;M_0^t(S(t),t)) & M_0^t(S(t),t) \ge K \end{cases}, \\ &H(t,S;N) = S\Phi(d_1) - Ne^{-r(T-t)}\Phi(d_2) + e^{-r(T-t)}\frac{S\sigma_1^2}{2r}\left(e^{r(T-t)}\Phi(d_1) - \left(\frac{S}{N}\right)^{\frac{-2r}{\sigma_1^2}}\Phi\left(d_1 - \frac{2r\sqrt{T-t}}{\sigma}\right)\right). \end{aligned} \tag{10}$$

$$
\begin{aligned}
d_1 &= \frac{\ln(S/N)+(r+\sigma_1^2/2)(T-t)}{\sigma_1\sqrt{T-t}},\\
d_2 &= \frac{\ln(S/N)+(r-\sigma_1^2/2)(T-t)}{\sigma_1\sqrt{T-t}}.
\end{aligned} \tag{11}
$$

**Fixed strike Lookback put option**

$$payoff \to MAX(K - m(S(T),T),0)$$

At the time $t$, the price would be $e^{-r(T-t)}E[\max(K-\min(m_0^t(S(t),t),m_t^T(S(T),T)),0)]$,

$$
\begin{aligned}
&For \quad K \le m_0^t(S(t),t)\\
&Eq. \to Ke^{-r(T-t)}\Phi(-d_2) - S\Phi(-d_1) + e^{-r(T-t)}\frac{S\sigma_1^2}{2r}\left(-e^{r(T-t)}\Phi(-d_1) - \left(\frac{S}{K}\right)^{\frac{-2r}{\sigma_1^2}}\Phi\left(-d_1+\frac{2r\sqrt{T-t}}{\sigma}\right)\right),\\
&For \quad K \ge m_0^t(S(t),t)\\
&Eq. \to e^{-r(T-t)}(K + m_0^t(S(t),t)(\Phi(d_2)-1)) - S\Phi(-d_1),\\
&\qquad + e^{-r(T-t)}\frac{S\sigma_1^2}{2r}\left(e^{r(T-t)}\Phi(-d_1) + \left(\frac{S}{m_0^t(S(t),t)}\right)^{\frac{-2r}{\sigma_1^2}}\Phi\left(-d_1+\frac{2r\sqrt{T-t}}{\sigma_1}\right)\right).
\end{aligned} \tag{12}
$$

$d_1$ and $d_2$ are the same as (11) formula.

# 3 | LOEX Option

In this section, we model and price the LOEX option using the concepts described in the previous section. As stated, this option is made by implementing the Lookback option on the Exchange option. The holder can replace an asset with the highest price from the start of the contract to maturity time with the lowest price of another asset in that period. Since the price of this option depends on the asset price path, this option is an exotic option.

To begin, we consider formula (1) and the following symbols:

$$
\begin{aligned}
M(S(t),t) &= \max_{0\le\omega\le t} S(\omega),\\
m(S^*(t),t) &= \min_{0\le\omega\le t} S^*(\omega).
\end{aligned} \tag{13}
$$

Let $M_t = \max_{0\le u\le t} S_u$ and $m_t^* = \min_{0\le u\le t} S_u^*$. Because the contract is path dependent, its Markov state is $(S,S^*,M,m)$, not merely $(S,S^*)$. Write its value as $G(t,S,S^*,M,m)$. The payoff is

$$G(T,S_T,S_T^*,M_T,m_T^*) = (M_T - m_T^*)^+. \tag{14}$$

Risk-neutral valuation gives

$$G(t,S,S^*,M,m) = e^{-r(T-t)}E^{Q}[(M_T-m_T^*)^+ \mid S_t = S, S_t^* = S^*, M_t = M, m_t^* = m]. \tag{15}$$

Away from the moving-extremum boundaries $S = M$ and $S^* = m$, Itô's formula gives

$$
\begin{aligned}
dG = &\left(G_t + rSG_S + rS^*G_{S^*} + \tfrac{1}{2}\sigma_1^2S^2G_{SS} + \tfrac{1}{2}\sigma_2^2(S^*)^2G_{S^*S^*} + \rho\sigma_1\sigma_2SS^*G_{SS^*}\right)dt\\
&+ \sigma_1SG_S\,dW_t^1 + \sigma_2S^*G_{S^*}\,dW_t^2.
\end{aligned} \tag{16}
$$

Now we use the Feynman–Kac formula:

$$\frac{\partial G}{\partial t} + rS\frac{\partial G}{\partial S} + rS^*\frac{\partial G}{\partial S^*} + \frac{1}{2}\sigma_1^2S^2\frac{\partial^2 G}{\partial S^2} + \rho\sigma_1\sigma_2SS^*\frac{\partial^2 G}{\partial S\,\partial S^*} + \frac{1}{2}\sigma_2^2(S^*)^2\frac{\partial^2 G}{\partial(S^*)^2} - rG = 0, \tag{17}$$

with $M$ and $m$ held as state variables in the interior and terminal condition

$$G(T, S, S^*, M, m) = (M - m)^+. \tag{18}$$

At the extremum boundaries the value must also satisfy the usual lookback compatibility conditions, formally $G_M = 0$ at $S = M$ and $G_m = 0$ at $S^* = m$ under smooth-fit regularity. Omitting $M$ and $m$ from the numerical state changes the contract and gives only a reduced-state illustration.

## 3.1 | PDE Solution by a Meshfree Method

Closed forms are generally unavailable for multi-asset path-dependent claims, so numerical approximation is required [14, 15]. We describe the meshfree radial-basis-function (RBF) method used for the supplied two-dimensional illustrations [16, 17]. For fixed running extrema $(M, m)$, truncate the asset domain to $[0, S_{\max}] \times [0, S^*_{\max}]$ and choose $J = N_1 N_2$ distinct collocation centers $z_i = (S_i, S_i^*)$.
With multiquadric basis functions

$$\varphi_i(S, S^*) = \sqrt{(S - S_i)^2 + (S^* - S_i^*)^2 + \varepsilon^2}, \tag{19}$$

the reduced-state approximation is

$$G(\tau, S, S^*; M, m) \approx \sum_{i=1}^{J} \lambda_i(\tau) \varphi_i(S, S^*), \qquad \tau = T - t. \tag{20}$$

The centers are essential: using one identical function for every coefficient would make the interpolation matrix rank deficient. Let $\mathbf{\Phi}_{ji} = \varphi_i(z_j)$ and let $\mathbf{L}_{ji} = (\mathcal{L}\varphi_i)(z_j)$, where

$$\mathcal{L} = rS\partial_S + rS^*\partial_{S^*} + \frac{1}{2}\sigma_1^2 S^2 \partial_{SS} + \rho\sigma_1\sigma_2 SS^* \partial_{SS^*} + \frac{1}{2}\sigma_2^2 (S^*)^2 \partial_{S^*S^*} - r. \tag{21}$$

Then the interior collocation system in time to maturity is

$$\mathbf{\Phi}\dot{\boldsymbol{\lambda}} = \mathbf{L}\boldsymbol{\lambda}. \tag{22}$$

For a time step $\Delta\tau$, the $\theta$-scheme is

$$(\mathbf{\Phi} - \theta\Delta\tau\, \mathbf{L})\, \boldsymbol{\lambda}^{k+1} = (\mathbf{\Phi} + (1 - \theta)\Delta\tau\, \mathbf{L})\, \boldsymbol{\lambda}^k. \tag{23}$$

The initial values at $\tau = 0$ are obtained from $(M - m)^+$. Boundary rows must be replaced by conditions consistent with the truncated domain and the extremum-state constraints. A complete LOEX solver must additionally discretize $M$ and $m$ or use an equivalent dimension-reduction argument; the supplied source does not provide that extension.

### 3.1.1 | Algorithm

1. Choose collocation centers and assemble $\mathbf{\Phi}$ and $\mathbf{L}$.
2. Fit $\boldsymbol{\lambda}^0$ to the terminal payoff $(M - m)^+$.
3. Replace the appropriate rows by boundary and extremum-compatibility conditions.
4. Advance the coefficient vector with the $\theta$-scheme.
5. Reconstruct $G = \mathbf{\Phi}\boldsymbol{\lambda}$ and repeat until $\tau = T$.

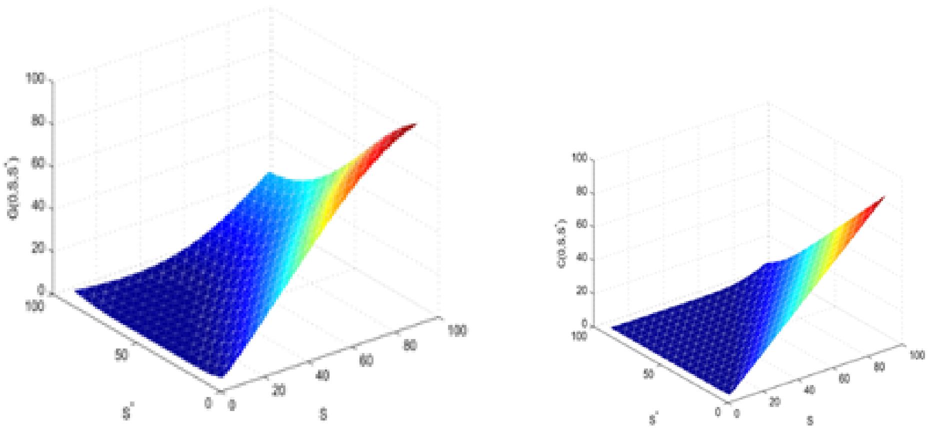
# 4 | Numerical Illustration

The supplied figures use $\sigma_1 = \sigma_2 = 0.125$, $\rho = -0.35$, $r = 0.05$, and $T = 1$. They compare a reduced two-dimensional LOEX surface with the Margrabe exchange-option surface. Because the MATLAB implementation and convergence diagnostics were not supplied, this section reports the figures as an exploratory illustration.

**Table 1.** Variables

| Variable | Value |
|---|---|
| $S_{max}, S^*_{max}$ | 100 |
| T | 1 |
| $N_1, N_2$ | 20 |
| $\Delta\tau$ | 0.01 |

Figure 1 displays the supplied LOEX and exchange-option price surfaces under the same parameter values.



**(a)** LOEX option price curve **(b)** Exchange option price curve

**Figure 1.** Option price curves

Figure 2 shows the pointwise difference between the two supplied surfaces.

The displayed LOEX values exceed the exchange-option values over much of the plotted domain, which is qualitatively consistent with the additional lookback feature. Figure 3 displays differences between the numerical first derivatives. These plots are descriptive; without code and grid-refinement results, they do not establish numerical accuracy.

# 5 | Conclusion

This paper formulates a LOEX payoff that combines exchange and lookback features. The corrected model requires the running maximum and minimum as state variables. A meshfree RBF discretization is described for fixed extrema, and the supplied plots are compared with a standard exchange-option surface. The additional path feature can increase contract value, but the magnitude shown by the plots cannot be treated as validated until the full extrema-state solver, boundary treatment, and convergence tests are supplied.

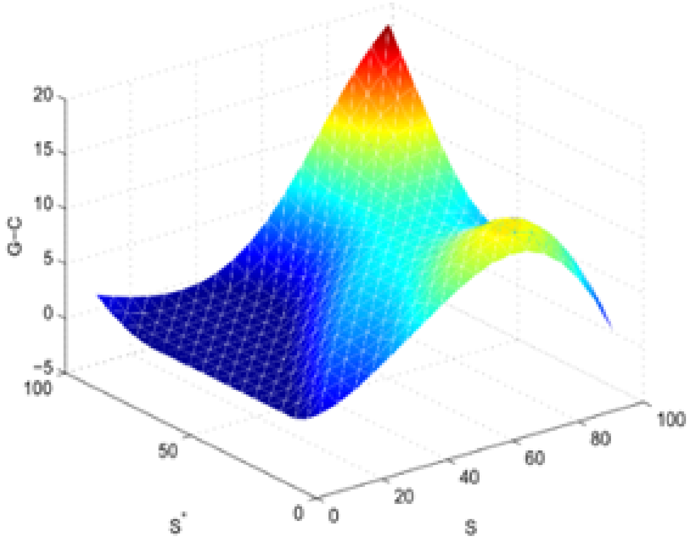
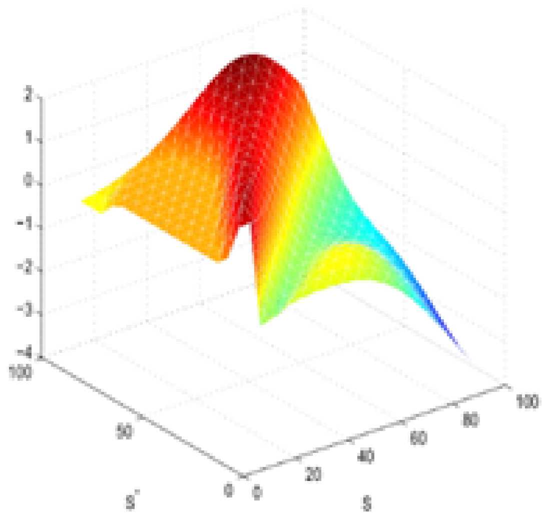
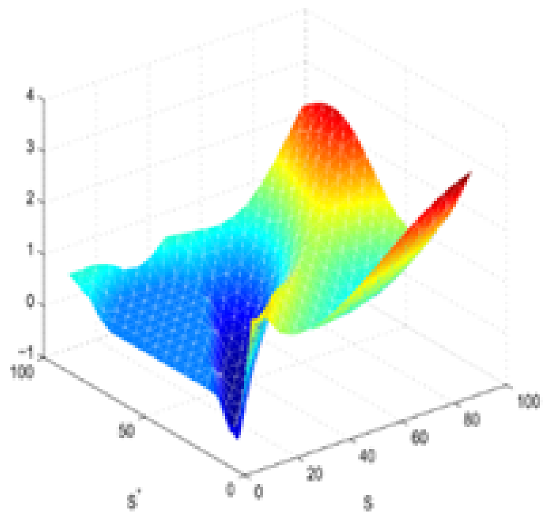



**Figure 2.** Difference between options

Furthermore, we can develop the model and make it more realistic for the market. For example, we can add components like credit risk and improve the performance of the price model.



(a) $\frac{\partial G}{\partial S} - \frac{\partial C}{\partial S}$



(b) $\frac{\partial G}{\partial S^*} - \frac{\partial C}{\partial S^*}$

**Figure 3.** Discrepancy of sensitivity of two options

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## Author Cintributions

All authors contributed equally.


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## Data Availability

The numerical figures were included in the supplied source package. The MATLAB code, spatial nodes, shape parameter, convergence study, and full extrema-state discretization were not supplied and should be archived before publication.

## Competing Interests

The competing-interests statement must be confirmed by all listed authors before publication.

## Generative-AI Assistance

Generative AI assisted with language, LaTeX preparation and technical representation of the paper.

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