



Paper Type: Original Article

Cost Reduction and Productivity Improvement in Mixed-Model Assembly Lines Through Workstation Optimization

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Citation:

Received: 10 May 2024

Revised: 26 July 2024

Accepted: 22 December 2024

Tanhaie, F. (2025). Cost reduction and productivity improvement in mixed-model assembly lines through workstation optimization. *Transactions on soft computing*, 1(1), 36-45.

Abstract

Assembly line balancing is a critical factor in the design and optimization of production systems, especially in multi-product environments. This paper presents a multi-objective mathematical model for mixed-model assembly line balancing, aiming to simultaneously minimize the number of workstations and the total equipment cost. The model incorporates precedence constraints, cycle time limitations, and task assignment rules. A computational experiment is conducted using the optimization software to validate the model's performance on a numerical example. Results demonstrate that the proposed approach efficiently reduces both the required number of stations and associated costs within a short computational time. This study contributes to the field by integrating cost efficiency into classical balancing frameworks, offering a practical solution for enhancing productivity and resource allocation in modern manufacturing settings.

Keywords: Mixed-model assembly line balancing, Linear programming, Multi-objective optimization, Equipment cost reduction.

1 | Introduction

Assembly lines are a special type of linear production system that plays a crucial role in industries that manufacture standardized products at high volumes. An assembly line is designed to combine parts and components through a set of required operations to produce the final product. In this context, the entire assembly process is decomposed into work elements. A work element is defined as the smallest unit of a productive task. In an assembly line, each workstation is assigned a subset of these work elements.

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doi: <https://doi.org/10.48314/tsc.vi.37>



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Throughout the assembly process, parts and workstations are connected sequentially via material handling systems such as conveyor belts. At each station, specific tasks are performed by an operator according to a fixed cycle time. As a result, the product moves progressively from one station to the next, and upon exiting the last station, it becomes the final output. Therefore, one of the key challenges is how to assign these tasks efficiently to workstations. Assembly line balancing problems have been extensively studied in the literature, with a focus on two main categories: As depicted in Fig. 1, the Simple Assembly Line Balancing Problem (SALBP) and the General Assembly Line Balancing Problem (GALBP).

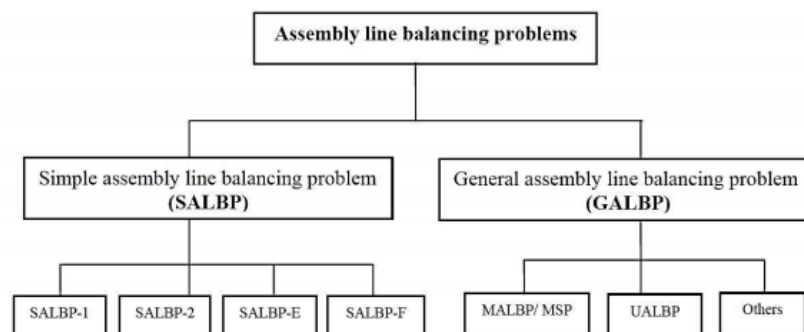


Fig. 1. Assembly line balancing problems.

Initially, assembly lines were developed with the aim of achieving cost efficiency through mass production of a single standardized product. These types of lines, known as single-model assembly lines, represent the simplest form of assembly systems, producing only one specific product type.

Assembly line problems generally involve assigning assembly tasks to stations while respecting precedence constraints among the tasks [1]. Researchers in this field have primarily focused on two main categories of problems. In the first category, the cycle time of the assembly line is given as input, and the objective function aims to minimize the number of required workstations [2]. In the second category, the number of workstations is fixed, and the goal is to minimize the cycle time [3]. An important point to consider is that this lack of variety often leads to insufficient demand for individual items to justify a dedicated assembly line. Therefore, organizations aiming to achieve high production flexibility—while maintaining the benefits of efficient flow production—require mixed-model assembly lines [4].

Precedence relations are constraints that define the required order of operations on a product. Due to technical and organizational conditions, these precedence constraints must be taken into account. Precedence relationships are typically represented in the form of a precedence graph, where nodes represent tasks and their durations, and arcs denote the precedence relationships between them. An example of a precedence graph consisting of nine tasks is illustrated in Fig. 2.

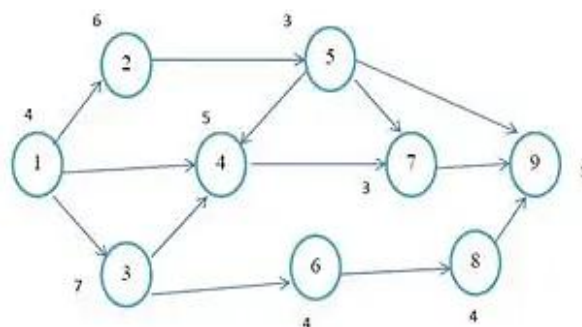


Fig. 2. Precedence relationships.

The mathematical formulation of the assembly line balancing problem was first introduced in 1955 by Salveson [5]. Over the past few decades, extensive research has been conducted on various aspects of assembly line problems. Different variants of the problem have been proposed in the literature based on line configuration—such as two-sided lines, U-shaped lines, and straight or parallel lines—as well as variations in product types, including single-model and mixed-model production environments.

As previously mentioned, research in this field initially focused on simple production lines and has since expanded to include more complex configurations such as mixed-model lines, stochastic task time lines, U-shaped lines, two-sided lines, robotic lines, and lines with parallel stations and equipment variability. Most studies have concentrated on two primary types of assembly line balancing problems. In the first type, the cycle time is given as input, and the objective is to minimize the number of workstations [6]. In the second type, the number of workstations is fixed, and the goal is to minimize the cycle time [7].

From a computational complexity perspective, the assembly line balancing problem falls into the class of Non-deterministic Polynomial-time Hard (NP-Hard) problems [8], [9], meaning that finding an optimal solution within a reasonable time frame is often challenging or even impractical [10]. Kim et al. and other researchers have applied heuristic and metaheuristic methods to solve this type of problem efficiently [11], [12], [13].

The main objective of this study is to minimize the total equipment cost of the assembly line by selecting appropriate tools and assigning tasks to workstations under a given cycle time. It is assumed that multiple types of equipment, each associated with different costs and processing times, are available for each task. At each workstation, only one type of equipment can be selected. Therefore, the model must determine which equipment to assign to each station and how to allocate tasks to workstations such that the total equipment acquisition cost is minimized. To achieve this, an exact branch-and-bound algorithm is employed to find optimal solutions, while a heuristic algorithm is also proposed to address larger instances with reduced computational time. The model is primarily formulated as a single-objective optimization problem aimed at minimizing equipment costs, although equipment selection inherently involves conflicting multi-objective criteria.

In this research, two objectives are considered: Minimizing the total equipment cost and integrating equipment selection with a branch-and-cut-based design approach combined with multi-criteria decision-making techniques.

Initially, tasks are assigned to workstations using an equal-slot strategy, and then all feasible combinations of equipment for each workstation are generated through a branch-and-cut procedure.

A similar problem involving parallel workstations and equipment selection was studied by Bukchin et al. [14], where both minimizing the number of workstations and the total cost were investigated. The proposed model is presented as a special case of the equipment selection problem, assuming that task execution times may exceed the given cycle time. An exact branch-and-bound optimization algorithm is developed to find the optimal solution.

2 | Problem Definition

This study addresses a multi-objective optimization problem in the context of assembly line balancing. The model incorporates two distinct objectives:

Minimizing the total number of workstations and minimizing the overall equipment costs associated with workstation configurations. To optimally solve the assembly line balancing problem under alternative operation sequences, a linear mathematical programming model is proposed. This formulation enables the simultaneous resolution of two interrelated sub-problems:

A decision-making process for selecting the most suitable assembly sequence among available alternatives, and an assembly line balancing task, which involves assigning work elements to stations while respecting precedence constraints and cycle time limitations.

In order to capture all feasible assembly sequences within the model, complete precedence paths are employed. The model selects a unique path that defines both the precedence relationships and processing times of all required tasks, while also determining their assignment to workstations. This integrated approach ensures that both sequencing and resource allocation decisions are made in a unified framework.

Furthermore, the proposed model considers a semi-automated production environment, where different types of equipment can be selected for each station based on their cost and performance characteristics.

Key Assumptions: The following assumptions underlie the proposed mathematical model:

Single-product production: Only one product type is assembled on the line.

Predefined equipment options: All available equipment types and their specifications are known in advance. Each machine's cost includes both acquisition cost and operational cost.

Fixed precedence relations: The precedence constraints between assembly tasks are clearly defined.

Non-divisible tasks: Assembly operations cannot be split into smaller units.

Deterministic task durations: Task times are fixed but may vary depending on the selected equipment.

Flexible task assignment: Any task can be assigned to any workstation, provided that the assigned equipment supports its execution and precedence constraints are respected.

Cycle time constraint: The total time allocated to each workstation must not exceed the predefined cycle time.

Incorporation of logistics times: Material handling, loading, and unloading times are included within the task durations.

A summary of notations, parameters, and decision variables used in the model is presented in *Table 1*. This includes indices, sets, input parameters, and variables involved in the formulation.

Table 1. Notation and parameters used in the model.

i, j	Indices for tasks and workstations, respectively
r	Index for alternative precedence paths ($r=1, 2, \dots, R$)
k	Index for equipment types

Parameters

N_{min}	Minimum number of required workstations
N_{max}	Upper bound on the number of workstations
C_{max}	Maximum allowable cycle time
P_r	Number of complete precedence paths
E_{ir}	Earliest workstation to which task i can be assigned when processed along path r
L_{ir}	Latest workstation to which task i can be assigned when processed along path r
T_{ir}	Set of immediate predecessors of task i when processed along path r
M_{jr}	Set of tasks that can be assigned to workstation j when processed along path r
E_{ck}	Cost of equipment type k ($k=1, \dots, K$)

Decision Variables

X_{ijr}	Binary variable; equals 1 if task i is assigned to workstation j along path r , and 0 otherwise
Y_j	Binary variable; equals 1 if workstation j is used, and 0 otherwise
Y_{jk}	Binary variable; equals 1 if equipment type k is assigned to workstation j , and 0 otherwise

After introducing the parameters and decision variables of the problem, we now proceed to present the mathematical model of the problem.

$$\text{MIN } Z1 = \sum_{j=m_{\min}+1}^{m_{\max}} j \cdot Y_j. \quad (1)$$

$$\text{MIN } Z2 = \sum_{j=1}^r \sum_{k=1}^K EC_j \cdot Y_{jk}. \quad (2)$$

$$\sum_{r=1}^{nr} \sum_{j=E_{ir}}^{L_{ir}} X_{ijr} = 1, \quad \text{for all } i. \quad (3)$$

$$\sum_{r=1}^{nr} \sum_{i \in T_{jr}} t_{ir} \cdot X_{ijr} \leq C_{\max}, \quad J = 1, 2, \dots, m_{\min}. \quad (4)$$

$$\sum_{r=1}^{nr} \sum_{i \in T_{jr}} t_{ir} \cdot X_{ijr} \leq C_{\max} \cdot Y_j, \quad J = m_{\min} + 1, \dots, m_{\max}. \quad (5)$$

$$\sum_{j=E_{pr}}^{L_{pr}} j \cdot X_{pjr} \leq \sum_{j=E_{ir}}^{L_{ir}} j \cdot X_{ijr}, \quad \text{for all } r, \text{ for all } i, \text{ for all } p \in PD_{ir}. \quad (6)$$

$$\sum_{j=E_{1r}}^{L_{1r}} X_{1jr} \leq \sum_{j=E_{ir}}^{L_{ir}} X_{ijr}, \quad \text{for all } r: r = 2, \dots, n. \quad (7)$$

$$\sum_{j=E_{1r}}^{L_{1r}} X_{1jr} \leq \sum_{j=E_{ir}}^{L_{ir}} X_{ijr}, \quad \text{for all } i, \text{ for all } r, \text{ for all } j \in [E_{ir}, L_{ir}]. \quad (8)$$

$$Y_j \in \{0, 1\}, \quad J = m_{\min} + 1, \dots, m_{\max}. \quad (9)$$

The mathematical model incorporates a set of constraints to ensure feasibility and optimality in task assignment, precedence relations, and resource allocation:

Task assignment constraint: This constraint guarantees that each task is assigned exactly to one workstation, ensuring full coverage of all required operations across the line.

Cycle time constraint: It ensures that the total processing time at each workstation does not exceed the given cycle time, maintaining line balance and throughput efficiency.

Precedence constraints: These constraints enforce the correct sequence of operations, ensuring that no task is started before its immediate predecessors are completed.

Path consistency constraint: This ensures that all tasks follow a single, coherent precedence path, preventing inconsistencies in the selected assembly sequence.

Binary decision variables: The model uses binary variables to represent discrete decisions, such as whether a workstation is activated or which equipment type is assigned to a station. These constraints collectively ensure

that the solution remains valid within the operational and technical boundaries of the assembly system, while optimizing both the number of workstations and the total equipment cost.

Problem objectives and computational results

The primary goal of the assembly line balancing problem is to assign tasks along an assembly line to workstations in such a way that:

The total number of required workstations is minimized, the idle time at each workstation is reduced as much as possible, Precedence constraints between tasks are satisfied, and the desired production output—determined by strategic managerial decisions—is achieved.

In line with these general objectives, the first aim of our proposed model is to minimize the total number of workstations, while the second objective focuses on minimizing overall equipment costs. As observed, both objectives align with the broader goals of effective assembly line design and optimization.

To demonstrate the performance and applicability of the proposed mathematical model, a representative numerical example has been selected based on standard cases from the literature. The detailed characteristics of this example are summarized in *Table 2*. The proposed model has been implemented on this set of test instances, and the computational results are presented in *Table 3*. These results illustrate the effectiveness of the model in simultaneously optimizing both objectives—workstation count and equipment cost—while respecting all operational constraints.

3 | Sample Test Problems for Assembly Line Balancing

In order to evaluate the performance of the proposed mathematical model and solution approach, two representative test cases have been selected from standard examples in the literature. These examples include task precedence constraints and processing times for each activity, as summarized in *Table 2*:

Table 2. Task precedence constraints and processing times.

	Processing time (Sample Problem 1)	Precedence	Processing time (Sample Problem 2)	Precedence
1	12	—	102	—
2	60	—	34	1
3	54	1	36	1
4	24	1	32	2
5	54	1	33	3
6	48	2	20	4, 5
7	6	4	130	4, 5
8	6	5	21	6
9	42	5	36	8
10	72	6, 7	78	7
11	60	8, 9	20	7, 10
12	48	10	78	7, 9
13	24	10	21	11
14	12	11	36	12, 15
15	36	12, 13	36	13
16	36	14	24	14
17	90	15, 16	—	15, 16
18	18	17	—	—
19	48	17	—	—
20	54	18, 19	—	—

In this model, achieving the defined objectives under the given constraints initially required several hours of computational time. However, with minor modifications to the formulation, we were able to obtain a high-quality solution in less than 10 minutes, significantly improving the model's efficiency.

Table 3. Computational results of the proposed model on sample problems.

	Time	Tasks	Machine Idle	Time	Tasks	Machine Idle
1	60	2	0	102	1	0
2	54	5, 8	6	66	2, 4	36
3	60	5, 8	0	199	3, 5, 7	1
4	60	3	0	77	6, 8, 9	25
5	54	11	6	78	10	24
6	174	4, 6, 7, 10	6	77	11, 13, 15	25
7	60	12, 14	0	78	12	24
8	180	15, 16, 17, 18	0	60	14, 16	42
9	48	19	12	—	—	—
10	54	20	6	—	—	—

The table above summarizes the computational results obtained by applying the proposed mathematical model to two sample problems. Each row represents a workstation, along with its total processing time, assigned tasks, and machine idle time.

In sample Problem 1, the model successfully assigns all 20 tasks to 10 workstations while respecting precedence constraints and cycle time limitations. The total idle time across all stations is relatively low, indicating efficient utilization of available resources. For sample Problem 2, which includes more complex precedence relationships and task durations, the model achieves a balanced solution using eight workstations. Although some stations exhibit higher idle times due to task duration mismatches, the overall efficiency remains acceptable.

4 | Performance Evaluation and Comparative Analysis

To further assess the effectiveness of the proposed model in handling multi-objective assembly line balancing under alternative precedence relations, an extended computational experiment was conducted. The model was implemented using the LINGO optimization software, and a representative numerical example was solved to evaluate its performance. This test case involves a hypothetical system consisting of 45 work elements, with a cycle time of 56 units and multiple alternative precedence paths. A detailed description of the problem parameters—including the number of tasks, cycle time, and the number of complete precedence paths—is provided in *Table 4*.

Table 4. Input parameters for the test problem.

Number of tasks	Cycle time	Number of complete paths	Number of partial paths
45	56	12	7
Number of constraints	Number of variables		
1383	10,840		

5 | Sensitivity Analysis

To further evaluate the robustness and practical applicability of the proposed multi-objective assembly line balancing model, a comprehensive sensitivity analysis was conducted. This analysis aims to investigate how changes in key input parameters affect the optimal values of the objective functions—namely, the total

number of workstations and the total equipment cost. We analyzed the effect of varying the cycle time from 50 to 70 units, in increments of 5 units, using the test case presented in *Table 5*.

Table 5. Changes in the parameter of the cycle time.

Cycle Time	Number of Workstations	Total Cost
50	12	28,500
55	10	26,300
60	9	24,750
65	8	23,900
70	7	23,200

The results indicate that as the cycle time increases, both the required number of workstations and the total equipment cost decrease. This is expected, since a higher cycle time allows more tasks to be assigned to each workstation, reducing the need for additional stations and associated equipment costs *Fig. 3*.

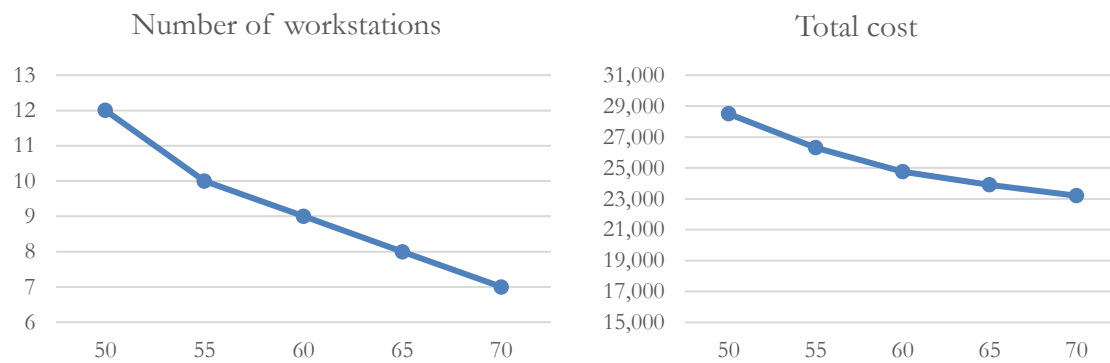


Fig. 3. Changes in the objective function and workstations.

This behavior confirms that cycle time has a significant influence on the structure of the solution and should be carefully considered during line design and optimization.

6 | Conclusion

An assembly line is a flow-based production system in which a product unit sequentially moves through workstations to be completed. If the total processing time across stations is not well-balanced, some stations may experience high idle times while others become bottlenecks, reducing overall efficiency. To address these issues, it is essential to balance the workload among stations with respect to the given cycle time. In this study, after identifying the key constraints and operational requirements, a binary integer programming model was formulated and solved multiple times to optimize two main objectives: Minimizing the number of required workstations and minimizing the total equipment cost across the line.

A numerical example was developed based on standard cases from the literature to demonstrate the effectiveness of the proposed model. The results confirmed that the model can efficiently allocate tasks to workstations while satisfying precedence constraints and optimizing resource usage.

This research presents a novel and distinct approach compared to previous studies, offering significant potential for future research directions. Among the possible extensions are:

Introducing cycle time minimization as a third objective function thereby increases the flexibility and adaptability of the model.

Developing heuristic or metaheuristic algorithms to reduce computational time and improve solution scalability for large-scale instances.

Incorporating additional real-world constraints, such as workforce limitations, parallel workstation configurations, alternative process plans, and spatial zoning restrictions, which were not considered in the current formulation.

Exploring alternative line configurations, including U-shaped and two-sided assembly lines, as potential variants of the problem for future investigation.

Extending the model to account for uncertainty in task times or demand patterns by formulating it under stochastic or fuzzy environments.

These suggested improvements and extensions provide promising avenues for further research aimed at enhancing the practical applicability of assembly line balancing models in complex manufacturing environments.

Acknowledgments

The author sincerely thanks the anonymous reviewers and the editorial team of Transactions on Soft Computing for their valuable comments, constructive suggestions, and careful evaluation of the manuscript. Their efforts greatly contributed to improving the clarity, quality, and overall presentation of this work.

Funding

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

Data Availability

The study is based on theoretical and computational analysis. All equations, algorithms, and methodological details are fully described in the article. Additional materials can be provided by the corresponding author upon reasonable request.

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